CHAPTER 7
EXPONENTS AND RADICALS

The operation of raising a number to a power is a special case of multiplication in which the factors are all equal. In examples such as $4^2 = 4 \times 4 = 16$ and $5^3 = 5 \times 5 \times 5 = 125$, the number 16 is the second power of 4 and the number 125 is the third power of 5. The expression 5^3 means that three 5's are to be multiplied together. Similarly, 4^2 means 4×4. The first power of any number is the number itself. The power is the number of times the number itself is to be taken as a factor.

The process of finding a root is the inverse of raising a number to a power. A root is a special factor of a number, such as 4 in the expression $4^2 = 16$. When a number is taken as a factor two times, as in the expression $4 \times 4 = 16$, it is called a square root. Thus, 4 is a square root of 16. By the same reasoning, 2 is a cube root of 8, since $2 \times 2 \times 2$ is equal to 8. This relationship is usually written as $2^3 = 8$.

POWERS AND ROOTS

A power of a number is indicated by an EXPONENT, which is a number in small print placed to the right and toward the top of the number. Thus, in $4^3 = 64$, the number 3 is the EXPONENT of the number 4. The exponent 3 indicates that the number 4, called the BASE, is to be raised to its third power. The expression is read "4 to the third power (or 4 cubed) equals 64." Similarly, $5^2 = 25$ is read "5 to the second power (or 5 squared) equals 25." Higher powers are read according to the degree indicated; for example, "fourth power," "fifth power," etc.

When an exponent occurs, it must always be written unless its value is 1. The exponent 1 usually is not written, but is understood. For example, the number 5 is actually 5^1. When we work with exponents, it is important to remember that any number that has no written exponent really has an exponent equal to 1.

A root of a number can be indicated by placing a radical sign, $\sqrt{\ }$, over the number and showing the root by placing a small number within the notch of the radical sign. Thus, $\sqrt[3]{64}$ indicates the cube root of 64, and $\sqrt[5]{32}$ indicates the fifth root of 32. The number that indicates the root is called the INDEX of the root. In the case of the square root, the index, 2, usually is not shown. When a radical has no index, the square root is understood to be the one desired. For example, $\sqrt{36}$ indicates the square root of 36. The line above the number whose root is to be found is a symbol of grouping called the vinculum. When the radical symbol is used, a vinculum, long enough to extend over the entire expression whose root is to be found, should be attached.

Practice problems. Raise to the indicated power or find the root indicated.

1. 2^3	2. 6^2	3. 4^3	4. 25^3
5. $\sqrt{16}$	6. $\sqrt[3]{8}$	7. $\sqrt[3]{125}$	8. $\sqrt[5]{32}$

Answers:

1. 8	2. 36	3. 64	4. 15,625
5. 4	6. 2	7. 5	8. 2

NEGATIVE INTEGERS

Raising to a power is multiplication in which all the numbers being multiplied together are equal. The sign of the product is determined, as in ordinary multiplication, by the number of minus signs. The number of minus signs is odd or even, depending on whether the exponent of the base is odd or even. For example, in the problem

$$(-2)^3 = (-2)(-2)(-2) = -8$$

there are three minus signs. The result is negative. In

$$(-2)^6 = 64$$

there are six minus signs. The result is positive.

Thus, when the exponent of a negative number is odd, the power is negative; when the exponent is even, the power is positive.

As other examples, consider the following:

$$(-3)^4 = 81$$

$$\left(-\frac{2}{5}\right)^3 = -\frac{8}{125}$$

$$(-2)^8 = 256$$

$$(-1)^5 = -1$$

Positive and negative numbers belong to the class called REAL NUMBERS. The square of a real number is positive. For example, $(-7)^2 = 49$ and $7^2 = 49$. The expression $(-7)^2$ is read "minus seven squared." Note that either seven squared or minus seven squared gives us +49. We cannot obtain -49 or any other negative number by squaring any real number, positive or negative.

Since there is no real number whose square is a negative number, it is sometimes said that the square root of a negative number does not exist. However, an expression under a square root sign may take on negative values. While the square root of a negative number cannot actually be found, it can be indicated.

The indicated square root of a negative number is called an IMAGINARY NUMBER. The number $\sqrt{-7}$, for example, is said to be imaginary. It is read "square root of minus seven." Imaginary numbers are discussed in chapter 15 of this course.

FRACTIONS

We recall that the exponent of a number tells the number of times that the number is to be taken as a factor. A fraction is raised to a power by raising the numerator and the denominator separately to the power indicated. The expression $\left(\frac{3}{7}\right)^2$ means $\frac{3}{7}$ is used twice as a factor. Thus,

$$\left(\frac{3}{7}\right)^2 = \frac{3}{7} \times \frac{3}{7} = \frac{3^2}{7^2}$$

$$= \frac{9}{49}$$

Similarly,

$$\left(-\frac{1}{5}\right)^2 = \frac{1}{25}$$

Since a minus sign can occupy any one of three locations in a fraction, notice that evaluating $\left(-\frac{1}{5}\right)^2$ is equivalent to

$$(-1)^2 \left(\frac{1}{5}\right)^2 \text{ or } \frac{(-1)^2}{5^2} \text{ or } \frac{1^2}{(-5)^2}$$

The process of taking a root of a number is the inverse of the process of raising the number to a power, and the method of taking the root of a fraction is similar. We may simply take the root of each term separately and write the result as a fraction. Consider the following examples:

1. $$\sqrt{\frac{36}{49}} = \frac{\sqrt{36}}{\sqrt{49}} = \frac{6}{7}$$

2. $$\sqrt[3]{\frac{8}{125}} = \frac{\sqrt[3]{8}}{\sqrt[3]{125}} = \frac{2}{5}$$

Practice problems. Find the values for the indicated operations:

1. $\left(\frac{1}{3}\right)^2$	2. $\left(\frac{3}{4}\right)^2$	3. $\left(\frac{6}{5}\right)^2$	4. $\left(\frac{2}{3}\right)^3$
5. $\sqrt{\frac{16}{36}}$	6. $\sqrt{\frac{16}{25}}$	7. $\sqrt[3]{\frac{8}{27}}$	8. $\sqrt{\frac{9}{49}}$

Answers:

1. 1/9	2. 9/16	3. 36/25	4. 8/27
5. 4/6	6. 4/5	7. 2/3	8. 3/7

DECIMALS

When a decimal is raised to a power, the number of decimal places in the result is equal to the number of places in the decimal multiplied by the exponent. For example, consider $(0.12)^3$. There are two decimal places in 0.12 and 3 is the exponent. Therefore, the number of places in the power will be 3(2) = 6. The result is as follows:

$$(0.12)^3 = 0.001728$$

The truth of this rule is evident when we recall the rule for multiplying decimals. Part of the rule states: Mark off as many decimal places in the product as there are decimal places in the factors together. If we carry out the multiplication, (0.12) x (0.12) x (0.12), it is obvious that there are six decimal places in the three factors together. The rule can be shown for any decimal raised to any power by simply carrying out the multiplication indicated by the exponent.

Consider these examples:

$$(1.4)^2 = 1.96$$
$$(0.12)^2 = 0.0144$$
$$(0.4)^3 = 0.064$$
$$(0.02)^2 = 0.0004$$
$$(0.2)^2 = 0.04$$

Finding a root of a number is the inverse of raising a number to a power. To determine the number of decimal places in the root of a perfect power, we divide the number of decimal places in the radicand by the index of the root. Notice that this is just the opposite of what was done in raising a number to a power.

Consider $\sqrt{0.0625}$. The square root of 625 is 25. There are four decimal places in the radicand, 0.0625, and the index of the root is 2. Therefore, $4 \div 2 = 2$ is the number of decimal places in the root. We have

$$\sqrt{0.0625} = 0.25$$

Similarly,

$$\sqrt{1.69} = 1.3$$
$$\sqrt[3]{0.027} = 0.3$$
$$\sqrt[3]{1.728} = 1.2$$
$$\sqrt[4]{0.0001} = 0.1$$

LAWS OF EXPONENTS

All of the laws of exponents may be developed directly from the definition of exponents. Separate laws are stated for the following five cases:

1. Multiplication.
2. Division.
3. Power of a power.
4. Power of a product.
5. Power of quotient.

MULTIPLICATION

To illustrate the law of multiplication, we examine the following problem:

$$4^3 \times 4^2 = ?$$

Recalling that 4^3 means 4 x 4 x 4 and 4^2 means 4 x 4, we see that 4 is used as a factor five times. Therefore $4^3 \times 4^2$ is the same as 4^5. This result could be written as follows:

$$4^3 \times 4^2 = 4 \times 4 \times 4 \times 4 \times 4$$
$$= 4^5$$

Notice that three of the five 4's came from the expression 4^3, and the other two 4's came from the expression 4^2. Thus we may rewrite the problem as follows:

$$4^3 \times 4^2 = 4^{(3+2)}$$
$$= 4^5$$

The law of exponents for multiplication may be stated as follows: To multiply two or more powers having the same base, add the exponents and raise the common base to the sum of the exponents. This law is further illustrated by the following examples:

$$2^3 \times 2^4 = 2^7$$
$$3 \times 3^2 = 3^3$$
$$15^4 \times 15^2 = 15^6$$
$$10^2 \times 10^{0.5} = 10^{2.5}$$

Common Errors

It is important to realize that the base must be the same for each factor, in order to apply the laws of exponents. For example, $2^3 \times 3^2$ is neither 2^5 nor 3^5. There is no way to apply the law of exponents to a problem of this kind. Another common mistake is to multiply the bases together. For example, this kind of error in the foregoing problem would imply that $2^3 \times 3^2$ is equivalent to 6^5, or 7776. The error of this may be proved as follows:

$$2^3 \times 3^2 = 8 \times 9$$
$$= 72$$

DIVISION

The law of exponents for division may be developed from the following example:

$$6^7 \div 6^5 = \frac{\cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6} \times 6 \times 6}{\cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6}}$$
$$= 6^2$$

Cancellation of the five 6's in the divisor with five of the 6's in the dividend leaves only two 6's, the product of which is 6^2.

This result can be reached directly by noting that 6^2 is equivalent to $6^{(7-5)}$. In other words, we have the following:

$$\begin{aligned} 6^7 \div 6^5 &= 6^{(7-5)} \\ &= 6^2 \end{aligned}$$

Therefore the law of exponents for division is as follows: To divide one power into another having the same base, subtract the exponent of the divisor from the exponent of the dividend. Use the number resulting from this subtraction as the exponent of the base in the quotient.

Use of this rule sometimes produces a negative exponent or an exponent whose value is 0. These two special types of exponents are discussed later in this chapter.

POWER OF A POWER

Consider the example $(3^2)^4$. Remembering that an exponent shows the number of times the base is to be taken as a factor and noting in this case that 3^2 is considered the base, we have

$$(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$$

Also in multiplication we add exponents. Thus,

$$3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 = 3^{(2+2+2+2)} = 3^8$$

Therefore,

$$\begin{aligned} (3^2)^4 &= 3^{(4 \times 2)} \\ &= 3^8 \end{aligned}$$

The laws of exponents for the power of a power may be stated as follows: To find the power of a power, multiply the exponents. It should be noted that this case is the only one in which multiplication of exponents is performed.

POWER OF A PRODUCT

Consider the example $(3 \cdot 2 \cdot 5)^3$. We know that

$$(3 \cdot 2 \cdot 5)^3 = (3 \cdot 2 \cdot 5)(3 \cdot 2 \cdot 5)(3 \cdot 2 \cdot 5)$$

Thus 3, 2, and 5 appear three times each as factors, and we can show this with exponents as 3^3, 2^3, and 5^3. Therefore,

$$(3 \cdot 2 \cdot 5)^3 = 3^3 \cdot 2^3 \cdot 5^3$$

The law of exponents for the power of a product is as follows: The power of a product is equal to the product obtained when each of the original factors is raised to the indicated power and the resulting powers are multiplied together.

POWER OF A QUOTIENT

The law of exponents for a power of an indicated quotient may be developed from the following example:

$$\begin{aligned} \left(\frac{2}{3}\right)^3 &= \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \\ &= \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} \\ &= \frac{2^3}{3^3} \end{aligned}$$

Therefore,

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$$

The law is stated as follows: The power of a quotient is equal to the quotient obtained when the dividend and divisor are each raised to the indicated power separately, before the division is performed.

Practice problems. Raise each of the following expressions to the indicated power:

1. $(3^2 \cdot 2^3)^2$
2. $3^5 \div 3^2$
3. $\left(\frac{3 \cdot 2}{5 \cdot 6}\right)^3$
4. $(-3^2)^3$
5. $\frac{5^3}{5}$
6. $(3 \cdot 2 \cdot 7)^2$

Answers:

1. $3^4 \times 2^6 = 5{,}184$
2. 27
3. $\frac{1}{125}$
4. $[(-3)^2]^3 = 729$
5. 25
6. $9 \cdot 4 \cdot 49 = 1{,}764$

SPECIAL EXPONENTS

Thus far in this discussion of exponents, the emphasis has been on exponents which are positive integers. There are two types of exponents which are not positive integers, and two which are treated as special cases even though they may be considered as positive integers.

ZERO AS AN EXPONENT

Zero occurs as an exponent in the answer to a problem such as $4^3 \div 4^3$. The law of exponents for division states that the exponents are to be subtracted. This is illustrated as follows:

$$\frac{4^3}{4^3} = 4^{(3-3)} = 4^0$$

Another way of expressing the result of dividing 4^3 by 4^3 is to use the fundamental axiom which states that any number divided by itself is 1. In order for the laws of exponents to hold true in all cases, this must also be true when any number raised to a power is divided by itself. Thus, $4^3/4^3$ must equal 1.

Since $4^3/4^3$ has been shown to be equal to both 4^0 and 1, we are forced to the conclusion that $4^0 = 1$.

By the same reasoning,

$$\frac{5}{5} = 5^{1-1} = 5^0$$

Also,

$$\frac{5}{5} = 1$$

Therefore,

$$5^0 = 1$$

Thus we see that any number divided by itself results in a 0 exponent and has a value of 1. By definition then, any number (other than zero) raised to the zero power equals 1. This is further illustrated in the following examples:

$$3^0 = 1$$
$$400^0 = 1$$
$$0.02^0 = 1$$
$$\left(\frac{1}{5}\right)^0 = 1$$
$$(\sqrt{3})^0 = 1$$

ONE AS AN EXPONENT

The number 1 arises as an exponent sometimes as a result of division. In the example $\frac{5^3}{5^2}$ we subtract the exponents to get

$$5^{3-2} = 5^1$$

This problem may be worked another way as follows:

$$\frac{5^3}{5^2} = \frac{\cancel{5} \cdot \cancel{5} \cdot 5}{\cancel{5} \cdot \cancel{5}} = 5$$

Therefore,

$$5^1 = 5$$

We conclude that any number raised to the first power is the number itself. The exponent 1 usually is not written but is understood to exist.

NEGATIVE EXPONENTS

If the law of exponents for division is extended to include cases where the exponent of the denominator is larger, negative exponents arise. Thus,

$$\frac{3^2}{3^5} = 3^{2-5} = 3^{-3}$$

Another way of expressing this problem is as follows:

$$\frac{3^2}{3^5} = \frac{\cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3} \cdot 3 \cdot 3 \cdot 3} = \frac{1}{3^3}$$

Therefore,

$$3^{-3} = \frac{1}{3^3}$$

We conclude that a number N with a negative exponent is equivalent to a fraction having the following form: Its numerator is 1; its denominator is N with a positive exponent whose absolute value is the same as the absolute value of the original exponent. In symbols, this rule may be stated as follows:

$$N^{-a} = \frac{1}{N^a}$$

Also,

$$\frac{1}{N^{-a}} = N^a$$

The following examples further illustrate the rule:

$$5^{-1} = \frac{1}{5}$$

$$6^{-2} = \frac{1}{6^2}$$

$$4^{-12} = \frac{1}{4^{12}}$$

$$\frac{1}{3^{-2}} = 3^2$$

Notice that the sign of an exponent may be changed by merely moving the expression which contains the exponent to the other position in the fraction. The sign of the exponent is changed as this move is made. For example,

$$\frac{1}{10^{-2}} = 1 \div \frac{1}{10^2}$$

$$= 1 \times \frac{10^2}{1}$$

Therefore,

$$\frac{1}{10^{-2}} = \frac{10^2}{1}$$

By using the foregoing relationship, a problem such as $3 \div 5^{-4}$ may be simplified as follows:

$$\frac{3}{5^{-4}} = 3 \times \frac{1}{5^{-4}}$$

$$= 3 \times \frac{5^4}{1}$$

$$= 3 \times 5^4$$

FRACTIONAL EXPONENTS

Fractional exponents obey the same laws as do integral exponents. For example,

$$4^{1/2} \times 4^{1/2} = 4^{(1/2 + 1/2)}$$

$$= 4^{2/2}$$

$$= 4^1 = 4$$

Another way of expressing this would be

$$4^{1/2} \times 4^{1/2} = (4^{1/2})^2$$

$$= 4^{(1/2 \times 2)}$$

$$= 4^1 = 4$$

Observe that the number $4^{1/2}$, when squared in the foregoing example, produced the number 4 as an answer. Recalling that a square root of a number N is a number x such that $x^2 = N$, we conclude that $4^{1/2}$ is equivalent to $\sqrt{4}$. Thus we have a definition, as follows: A fractional exponent of the form $1/r$ indicates a root, the index of which is r. This is further illustrated in the following examples:

$$2^{1/2} = \sqrt{2}$$

$$4^{1/3} = \sqrt[3]{4}$$

$$6^{2/3} = (6^{1/3})^2 = (\sqrt[3]{6})^2$$

Also,

$$6^{2/3} = (6^2)^{1/3} = \sqrt[3]{36}$$

Notice that in an expression such as $8^{2/3}$ we can either find the cube root of 8 first or square 8 first, as shown by the following example:

$$(8^{1/3})^2 = 2^2 = 4 \text{ and } (8^2)^{1/3} = \sqrt[3]{64} = 4$$

All the numbers in the evaluation of $8^{2/3}$ remain small if the cube root is found before raising the number to the second power. This order of operation is particularly desirable in evaluating a number like $64^{5/6}$. If 64 were first raised to the fifth power, a large number would result. It would require a great deal of unnecessary effort to find the sixth root of 64^5. The result is obtained easily, if we write

$$64^{5/6} = (64^{1/6})^5 = 2^5 = 32$$

If an improper fraction occurs in an exponent, such as 7/3 in the expression $2^{7/3}$, it is customary to keep the fraction in that form rather than express it as a mixed number. In fraction form an exponent shows immediately what power is intended and what root is intended. However, $2^{7/3}$ can be expressed in another form and simplified by changing the improper fraction to a mixed number and writing the fractional part in the radical form as follows:

$$2^{7/3} = 2^{2 + 1/3} = 2^2 \cdot 2^{1/3} = 4\sqrt[3]{2}$$

The law of exponents for multiplication may be combined with the rule for fractional exponents to solve problems of the following type:

PROBLEM: Evaluate the expression $4^{2.5}$.

SOLUTION:

$$\begin{aligned} 4^{2.5} &= 4^2 \times 4^{0.5} \\ &= 16 \times 4^{1/2} \\ &= 16 \times 2 \\ &= 32 \end{aligned}$$

Practice problems:

1. Perform the indicated division: $\frac{2}{2^{1/3}}$
2. Find the product: $7^{2/5} \times 7^{1/10} \times 7^{3/10}$
3. Rewrite with a positive exponent and simplify: $9^{-1/2}$
4. Evaluate $100^{3/2}$
5. Evaluate $(8^0)^5$

Answers:

1. $2^{3/3} \div 2^{1/3} = \sqrt[3]{4}$
2. $7^{8/10}$
3. $\frac{1}{9^{1/2}} = \frac{1}{3}$
4. 1,000
5. 1

SCIENTIFIC NOTATION AND POWERS OF 10

Technicians, engineers, and others engaged in scientific work are often required to solve problems involving very large and very small numbers. Problems such as

$$\frac{22{,}684 \times 0.00189}{0.0713 \times 83 \times 7}$$

are not uncommon. Solving such problems by the rules of ordinary arithmetic is laborious and time consuming. Moreover, the tedious arithmetic process lends itself to operational errors. Also there is difficulty in locating the decimal point in the result. These difficulties can be greatly reduced by a knowledge of the powers of 10 and their use.

The laws of exponents form the basis for calculation using powers of 10. The following list includes several decimals and whole numbers expressed as powers of 10:

$$\begin{aligned} 10{,}000 &= 10^4 \\ 1{,}000 &= 10^3 \\ 100 &= 10^2 \\ 10 &= 10^1 \\ 1 &= 10^0 \\ 0.1 &= 10^{-1} \\ 0.01 &= 10^{-2} \\ 0.001 &= 10^{-3} \\ 0.0001 &= 10^{-4} \end{aligned}$$

The concept of scientific notation may be demonstrated as follows:

$$\begin{aligned} 60{,}000 &= 6.0000 \times 10{,}000 \\ &= 6 \times 10^4 \\ 538 &= 5.38 \times 100 \\ &= 5.38 \times 10^2 \end{aligned}$$

Notice that the final expression in each of the foregoing examples involves a number between 1 and 10, multiplied by a power of 10. Furthermore, in each case the exponent of the power of 10 is a number equal to the number of digits between the new position of the decimal point and the original position (understood) of the decimal point.

We apply this reasoning to write any number in scientific notation; that is, as a number between 1 and 10 multiplied by the appropriate power of 10. The appropriate power of 10 is found by the following mechanical steps:

1. Shift the decimal point to standard position, which is the position immediately to the right of the first nonzero digit.
2. Count the number of digits between the new position of the decimal point and its original position. This number indicates the value of the exponent for the power of 10.
3. If the decimal point is shifted to the left, the sign of the exponent of 10 is positive; if the decimal point is shifted to the right, the sign of the exponent is negative.

The validity of this rule, for those cases in which the exponent of 10 is negative, is demonstrated as follows:

$$0.00657 = 6.57 \times 0.001$$
$$= 6.57 \times 10^{-3}$$
$$0.348 = 3.48 \times 0.1$$
$$= 3.48 \times 10^{-1}$$

Further examples of the use of scientific notation are given as follows:

$$543{,}000{,}000 = 5.43 \times 10^{8}$$
$$186 = 1.86 \times 10^{2}$$
$$243.01 = 2.4301 \times 10^{2}$$
$$0.0000007 = 7 \times 10^{-7}$$
$$0.00023 = 2.3 \times 10^{-4}$$

Multiplication Using Powers of 10

From the law of exponents for multiplication we recall that to multiply two or more powers to the same base we add their exponents. Thus,

$$10^{4} \times 10^{2} = 10^{6}$$

We see that multiplying powers of 10 together is an application of the general rule. This is demonstrated in the following examples:

1. $10{,}000 \times 100 = 10^{4} \times 10^{2}$
$= 10^{4+2}$
$= 10^{6}$

2. $0.0000001 \times 0.001 = 10^{-7} \times 10^{-3}$
$= 10^{-7+(-3)}$
$= 10^{-10}$

3. $10{,}000 \times 0.001 = 10^{4} \times 10^{-3}$
$= 10^{4-3}$
$= 10$

4. $23{,}000 \times 500 = ?$
$23{,}000 = 2.3 \times 10^{4}$
$500 = 5 \times 10^{2}$

Therefore,

$$23{,}000 \times 500 = 2.3 \times 10^{4} \times 5 \times 10^{2}$$
$$= 2.3 \times 5 \times 10^{4} \times 10^{2}$$
$$= 11.5 \times 10^{6}$$
$$= 1.15 \times 10^{7}$$

5. $62{,}000 \times 0.0003 \times 4{,}600 = ?$
$62{,}000 = 6.2 \times 10^{4}$
$0.0003 = 3 \times 10^{-4}$
$4{,}600 = 4.6 \times 10^{3}$

Therefore,

$$62{,}000 \times 0.0003 \times 4{,}600 = 6.2 \times 3 \times 4.6 \times 10^{4} \times 10^{-4} \times 10^{3}$$
$$= 85.56 \times 10^{3}$$
$$= 8.556 \times 10^{4}$$

Practice problems. Multiply, using powers of 10. For the purposes of this exercise, treat all numbers as exact numbers:

1. $10{,}000 \times 0.001 \times 100$
2. $0.000350 \times 5{,}000{,}000 \times 0.0004$
3. $3{,}875 \times 0.000032 \times 3{,}000{,}000$
4. $7{,}000 \times 0.015 \times 1.78$

Answers:

1. 1.0×10^{3}
2. 7.0×10^{-1}
3. 3.72×10^{5}
4. 1.869×10^{2}

Division Using Powers of 10

The rule of exponents for division states that, for powers of the same base, the exponent of the denominator is subtracted from the exponent of the numerator. Thus,

$$\frac{10^{7}}{10^{3}} = 10^{7-3}$$
$$= 10^{4}$$

It should be remembered that powers may be transferred from numerator to denominator or from denominator to numerator by simply changing the sign of the exponent. The following examples illustrate the use of this rule for powers of 10:

1. $\dfrac{72{,}000}{0.0012} = \dfrac{7.2 \times 10^{4}}{1.2 \times 10^{-3}}$
$= \dfrac{7.2}{1.2} \times 10^{4} \times 10^{3}$
$= 6 \times 10^{7}$

2. $$\frac{44 \times 10^{-4}}{11 \times 10^{-5}} = \frac{44}{11} \times 10^{-4} \times 10^{5}$$
$$= 4 \times 10$$

Combined Multiplication and Division

Using the rules already shown, multiplication and division involving powers of 10 may be combined. The usual method of solving such problems is to multiply and divide alternately until the problem is completed. For example,

$$\frac{36{,}000 \times 1.1 \times 0.06}{0.012 \times 2{,}200}$$

Rewriting this problem in scientific notation, we have

$$\frac{3.6 \times 10^4 \times 1.1 \times 6 \times 10^{-2}}{1.2 \times 10^{-2} \times 2.2 \times 10^3} = \frac{3.6 \times 1.1 \times 6}{1.2 \times 2.2} \times 10$$
$$= 9 \times 10$$
$$= 90$$

Notice that the elimination of 0's, wherever possible, simplifies the computation and makes it an easy matter to place the decimal point.

SIGNIFICANT DIGITS.—One of the most important advantages of scientific notation is the fact that it simplifies the task of determining the number of significant digits in a number. For example, the fact that the number 0.00045 has two significant digits is sometimes obscured by the presence of the 0's. The confusion can be avoided by writing the number in scientific notation, as follows:

$$0.00045 = 4.5 \times 10^{-4}$$

Practice problems. Express the numbers in the following problems in scientific notation and round off before performing the calculation. In each problem, round off calculation numbers to one more digit than the number of significant digits in the least accurate number; round the answer to the number of significant digits in the least accurate number:

1. $$\frac{0.000063 \times 50.4 \times 0.007213}{780 \times 0.682 \times 0.018}$$

2. $$\frac{0.015 \times 216 \times 1.78}{72 \times 0.0624 \times 0.0353}$$

3. $$\frac{0.000079 \times 0.00036}{29 \times 10^{-8}}$$

Answers:

1. 2.4×10^{-6}
2. 3.6×10
3. 9.8×10^{-2}

Other Applications

The applications of powers of 10 may be broadened to include problems involving reciprocals and powers of products.

RECIPROCALS.—The following example illustrates the use of powers of 10 in the formation of a reciprocal:

$$\frac{1}{250{,}000 \times 300 \times 0.02}$$
$$= \frac{1}{2.5 \times 10^5 \times 3 \times 10^2 \times 2 \times 10^{-2}}$$
$$= \frac{10^{-5}}{2.5 \times 3 \times 2}$$
$$= \frac{10^{-5}}{15}$$

Rather than write the numerator as 0.00001, write it as the product of two factors, one of which may be easily divided, as follows:

$$\frac{10^{-5}}{15} = \frac{10^2 \times 10^{-7}}{15}$$
$$= \frac{100}{15} \times 10^{-7}$$
$$= 6.67 \times 10^{-7}$$
$$= 0.000000667$$

POWER OF A PRODUCT.—The following example illustrates the use of powers of 10 in finding the power of a product:

$$(80{,}000 \times 2 \times 10^5)^2 = (8 \times 10^4 \times 2 \times 10^5)^2$$
$$= 8^2 \times 2^2 \times (10^{4+5})^2$$
$$= 64 \times 4 \times 10^{18}$$
$$= 256 \times 10^{18}$$
$$= 2.56 \times 10^{20}$$

RADICALS

An expression such as $\sqrt{2}$, $\sqrt[3]{5}$, or $\sqrt{a + b}$ that exhibits a radical sign, is referred to as a

RADICAL. We have already worked with radicals in the form of fractional exponents, but it is also frequently necessary to work with them in the radical form. The word "radical" is derived from the Latin word "radix," which means "root." The word "radix" itself is more often used in modern mathematics to refer to the base of a number system, such as the base 2 in the binary system. However, the word "radical" is retained with its original meaning of "root."

The radical symbol ($\sqrt{}$) appears to be a distortion of the initial letter "r" from the word "radix." With long usage, the r gradually lost its significance as a letter and became distorted into the symbol as we use it. The vinculum helps to specify exactly which of the letters and numbers following the radical sign actually belong to the radical expression.

The number under a radical sign is the RADICAND. The index of the root (except in the case of a square root) appears in the trough of the radical sign. The index tells what root of the radicand is intended. For example, in $\sqrt[5]{32}$, the radicand is 32 and the index of the root is 5. The fifth root of 32 is intended. In $\sqrt{50}$, the square root of 50 is intended. When the index is 2, it is not written, but is understood.

If we can find one square root of a number we can always find two of them. Remember $(3)^2$ is 9 and $(-3)^2$ is also 9. Likewise $(4)^2$ and $(-4)^2$ both equal 16 and $(5)^2$ and $(-5)^2$ both equal 25. Conversely, $\sqrt{9}$ is +3 or -3, $\sqrt{16}$ is +4 or -4, and $\sqrt{25}$ is +5 or -5. When we wish to show a number that may be either positive or negative, we may use the symbol $\pm$ which is read "plus or minus." Thus ± 3 means "plus or minus 3." Usually when a number is placed under the radical sign, only its positive root is desired and, unless otherwise specified, it is the only root that need be found.

COMBINING RADICALS

A number written in front of another number and intended as a multiplier is called a COEFFICIENT. The expression 5x means 5 times x; ay means a times y; and $7\sqrt{2}$ means 7 times $\sqrt{2}$. In these examples, 5 is the coefficient of x, a is the coefficient of y, and 7 is the coefficient of $\sqrt{2}$.

Radicals having the same index and the same radicand are SIMILAR. Similar radicals may have different coefficients in front of the radical sign. For example, $3\sqrt{2}$, $\sqrt{2}$, and $\frac{1}{5}\sqrt{2}$ are similar radicals. When a coefficient is not written, it is understood to be 1. Thus, the coefficient of $\sqrt{2}$ is 1. The rule for adding radicals is the same as that stated for adding denominate numbers: Add only units of the same kind. For example, we could add $2\sqrt{3}$ and $4\sqrt{3}$ because the "unit" in each of these numbers is the same ($\sqrt{3}$). By the same reasoning, we could not add $2\sqrt{3}$ and $4\sqrt{5}$ because these are not similar radicals.

Addition and Subtraction

When addition or subtraction of similar radicals is indicated, the radicals are combined by adding or subtracting their coefficients and placing the result in front of the radical. Adding $3\sqrt{2}$ and $5\sqrt{2}$ is similar to adding 3 bolts and 5 bolts. The following examples illustrate the addition and subtraction of similar radical expressions:

1. $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$
2. $1/2\ (\sqrt[4]{3}) + 1/3\ (\sqrt[4]{3}) = 5/6\ (\sqrt[4]{3})$
3. $\sqrt{5} - 6\sqrt{5} + 2\sqrt{5} = -3\sqrt{5}$
4. $-5\sqrt[3]{7} - 2\sqrt[3]{7} + 7\sqrt[3]{7} = 0$

Example 4 illustrates a case that is sometimes troublesome. The sum of the coefficients, -5, -2, and 7, is 0. Therefore, the coefficient of the answer would be 0, as follows:

$$0(\sqrt[3]{7}) = 0 \times \sqrt[3]{7}$$

Thus the final answer is 0, since 0 multiplied by any quantity is still 0.

Practice problems. Perform the indicated operations:

1. $4\sqrt{3} - \sqrt{3} + 5\sqrt{3}$
2. $\frac{1}{2}\sqrt{6} + \sqrt{6}$
3. $\sqrt[3]{5} - 6\sqrt[3]{5}$
4. $-2\sqrt{10} - 7\sqrt{10}$

Answers

1. $8\sqrt{3}$
2. $\frac{3}{2}\sqrt{6}$
3. $-5\sqrt[3]{5}$
4. $-9\sqrt{10}$

Multiplication and Division

If a radical is written immediately after another radical, multiplication is intended. Sometimes a dot is placed between the radicals, but not always. Thus, either $\sqrt{7} \cdot \sqrt{11}$ or $\sqrt{7}\sqrt{11}$ means multiplication.

When multiplication or division of radicals is indicated, several radicals having the same index can be combined into one radical, if desired. Radicals having the same index are said to be of the SAME ORDER. For example, $\sqrt{2}$ is a radical of the second order. The radicals $\sqrt{2}$ and $\sqrt{5}$ are of the same order.

If radicals are of the same order, the radicands can be multiplied or divided and placed under one radical symbol. For example, $\sqrt{5}$ multiplied by $\sqrt{3}$ is the same as $\sqrt{5 \times 3}$. Also, $\sqrt{6}$ divided by $\sqrt{3}$ is the same as $\sqrt{6 \div 3}$. If coefficients appear before the radicals, they also must be included in the multiplication or division. This is illustrated in the following examples:

1.
$$\begin{aligned} 2\sqrt{2} \cdot 3\sqrt{5} &= 2 \cdot \sqrt{2} \cdot 3 \cdot \sqrt{5} \\ &= 2 \cdot 3\sqrt{2} \cdot \sqrt{5} \\ &= 2 \cdot 3\sqrt{2 \cdot 5} \\ &= 6\sqrt{10} \end{aligned}$$

2.
$$\begin{aligned} \frac{15\sqrt{6}}{3\sqrt{3}} &= \frac{15}{3} \times \sqrt{\frac{6}{3}} \\ &= 5 \times \sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

It is important to note that what we have said about multiplication and division does not apply to addition. A typical error is to treat the expression $\sqrt{9+4}$ as if it were equivalent to $\sqrt{9} + \sqrt{4}$. These expressions cannot be equivalent, since 3 + 2 is not equivalent to $\sqrt{13}$.

FACTORING RADICALS.—A radical can be split into two or more radicals of the same order if the radicand can be factored. This is illustrated in the following examples:

1. $\sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$
2. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2}$
$= \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$
3. $\dfrac{\sqrt{20}}{\sqrt{5}} = \dfrac{\sqrt{4} \cdot \sqrt{5}}{\sqrt{5}}$
$= \sqrt{4} = 2$

SIMPLIFYING RADICALS

Some radicals may be changed to an equivalent form that is easier to use. A radical is in its simplest form when no factor can be removed from the radical, when there is no fraction under the radical sign, and when the index of the root cannot be reduced. A factor can be removed from the radical if it occurs a number of times equal to the index of the root. The following examples illustrate this:

1. $\sqrt{28} = \sqrt{2^2 \cdot 7} = 2\sqrt{7}$
2. $\sqrt[3]{54} = \sqrt[3]{3^3 \cdot 2} = 3(\sqrt[3]{2})$
3. $\sqrt[5]{160} = \sqrt[5]{2^5 \cdot 5} = 2(\sqrt[5]{5})$

Removing a factor that occurs a number of times equal to the index of the root is equivalent to separating a radical into two radicals so that one radicand is a perfect power. The radical sign can be removed from the number that is a perfect square, cube, fourth power, etc. The root taken becomes the coefficient of the remaining radical.

In order to simplify radicals easily, it is convenient to know the squares of whole numbers up to about 25 and a few of the smaller powers of the numbers 2, 3, 4, 5, and 6. Table 7-1 shows some frequently used powers of numbers.

Table 7-1.—Powers of numbers.

$1^2 = 1$	$14^2 = 196$
$2^2 = 4$	$15^2 = 225$
$3^2 = 9$	$16^2 = 256$
$4^2 = 16$	$17^2 = 289$
$5^2 = 25$	$18^2 = 324$
$6^2 = 36$	$19^2 = 361$
$7^2 = 49$	$20^2 = 400$
$8^2 = 64$	$21^2 = 441$
$9^2 = 81$	$22^2 = 484$
$10^2 = 100$	$23^2 = 529$
$11^2 = 121$	$24^2 = 576$
$12^2 = 144$	$25^2 = 625$
$13^2 = 169$	

(A)

Table 7-1.—Powers of numbers—Continued.

(B)	(C)
$2^1 = 2$	$3^1 = 3$
$2^2 = 4$	$3^2 = 9$
$2^3 = 8$	$3^3 = 27$
$2^4 = 16$	$3^4 = 81$
$2^5 = 32$	$3^5 = 243$
$2^6 = 64$	
$2^7 = 128$	
$2^8 = 256$	

(D)	(E)	(F)
$4^1 = 4$	$5^1 = 5$	$6^1 = 6$
$4^2 = 16$	$5^2 = 25$	$6^2 = 36$
$4^3 = 64$	$5^3 = 125$	$6^3 = 216$
$4^4 = 256$	$5^4 = 625$	

Referring to table 7-1 (A), we see that the series of numbers

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

comprises all the perfect squares from 1 to 100 inclusive. If any one of these numbers appears under a square root symbol, the radical sign can be removed immediately. This is illustrated as follows:

$$\sqrt{25} = 5$$

$$\sqrt{81} = 9$$

A radicand such as 75, which has a perfect square (25) as a factor, can be simplified as follows:

$$\begin{aligned}\sqrt{75} &= \sqrt{25 \cdot 3}\\ &= \sqrt{25} \cdot \sqrt{3}\\ &= 5\sqrt{3}\end{aligned}$$

This procedure is further illustrated in the following problems:

1. $\sqrt{8} = \sqrt{4 \cdot 2}$
   $= \sqrt{4} \cdot \sqrt{2}$
   $= 2\sqrt{2}$
2. $\sqrt{72} = \sqrt{36 \cdot 2}$
   $= \sqrt{36} \cdot \sqrt{2}$
   $= 6\sqrt{2}$

By reference to the perfect fourth powers in table 7-1, we may simplify a radical such as $\sqrt[4]{405}$. Noting that 405 has the perfect fourth power 81 as a factor, we have the following:

$$\begin{aligned}\sqrt[4]{405} &= \sqrt[4]{81 \cdot 5}\\ &= \sqrt[4]{81} \cdot \sqrt[4]{5}\\ &= 3\,(\sqrt[4]{5})\end{aligned}$$

As was shown with fractional exponents, taking a root is equivalent to dividing the exponent of a power by the index of the root. If a factor of the radicand has an exponent that is not a multiple of the index of the root, the factor may be separated so that one exponent is divisible by the index, as in

$$\sqrt{3^7} = \sqrt{3^6 \cdot 3} = 3^{6/2} \cdot 3^{1/2} = 3^3 \cdot \sqrt{3} = 27\sqrt{3}$$

Consider also

$$\begin{aligned}\sqrt{2^3 \cdot 3^7 \cdot 5} &= \sqrt{2^2 \cdot 2 \cdot 3^6 \cdot 3 \cdot 5}\\ &= 2 \cdot 3^3(\sqrt{2 \cdot 3 \cdot 5})\\ &= 54\sqrt{30}\end{aligned}$$

If the radicand is a large number, the perfect powers that are factors are not always obvious. In such a case the radicand can be separated into prime factors. For example,

$$\begin{aligned}\sqrt{8{,}820} &= \sqrt{2^2 \cdot 3^2 \cdot 5 \cdot 7^2}\\ &= 2 \cdot 3 \cdot 7\sqrt{5}\\ &= 42\sqrt{5}\end{aligned}$$

Practice problems. Simplify the radicals and reduce to lowest terms:

1. $\dfrac{\sqrt{3} \cdot \sqrt{15}}{\sqrt{5}}$
2. $\dfrac{\sqrt[3]{81}}{\sqrt[3]{27}}$
3. $\dfrac{18(\sqrt[3]{30})}{3(\sqrt[3]{10})}$
4. $\dfrac{\sqrt{8{,}820}}{\sqrt{180}}$

Answers:

1. 3
2. $\sqrt[3]{3}$
3. $6(\sqrt[3]{3})$
4. 7

RATIONAL AND IRRATIONAL NUMBERS

Real and imaginary numbers make up the number system of algebra. Imaginary numbers are discussed in chapter 15 of this course. Real numbers are either rational or irrational. The word RATIONAL comes from the word "ratio." A number is rational if it can be expressed as the quotient, or ratio, of two whole numbers. Rational numbers include fractions like 2/7, whole numbers, and radicals if the radical sign is removable.

Any whole number is rational. Its denominator is 1. For instance, 8 equals $\frac{8}{1}$, which is the quotient of two integers. A number like $\sqrt{16}$ is rational, since it can be expressed as the quotient of two integers in the form $\frac{4}{1}$. The following are also examples of rational numbers:

1. $\sqrt{\frac{25}{9}}$, which equals $\frac{5}{3}$
2. -6, which equals $\frac{-6}{1}$
3. $5\frac{2}{7}$, which equals $\frac{37}{7}$

Any rational number can be expressed as the quotient of two integers in many ways. For example,

$$7 = \frac{7}{1} = \frac{14}{2} = \frac{21}{3} \ldots$$

An IRRATIONAL number is a real number that cannot be expressed as the ratio of two integers. The numbers $\sqrt{3}$, $5\sqrt{2}$, $\sqrt{7}$, $\frac{3}{8}\sqrt{20}$, and $\frac{2}{\sqrt{5}}$ are examples of irrational numbers.

Rationalizing Denominators

Expressions such as $\frac{7}{\sqrt{2}}$ and $\frac{\sqrt{2}}{5\sqrt{3}}$ have irrational numbers in the denominator. If the denominators are changed immediately to decimals, as in

$$\frac{7}{\sqrt{2}} = \frac{7}{1.4142}$$

the process of evaluating a fraction becomes an exercise in long division. Such a fraction can be evaluated quickly by first changing the denominator to a rational number. Converting a fraction with an irrational number in its denominator to an equivalent fraction with a rational number in the denominator is called RATIONALIZING THE DENOMINATOR.

Multiplying a fraction by 1 leaves the value of the fraction unchanged. Since any number divided by itself equals 1, it follows, for example, that

$$\frac{\sqrt{2}}{\sqrt{2}} = 1$$

If the numerator and denominator of $\frac{7}{\sqrt{2}}$ are each multiplied by $\sqrt{2}$, another fraction having the same value is obtained. The result is

$$\frac{7}{\sqrt{2}} = \frac{7}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$$

The denominator of the new equivalent fraction is 2, which is rational. The decimal value of the fraction is

$$\frac{7\sqrt{2}}{2} = \frac{7(1.4142)}{2} = 7(0.7071) = 4.9497$$

To rationalize the denominator in $\frac{\sqrt{2}}{5\sqrt{3}}$ we multiply the numerator and denominator by $\sqrt{3}$. We get

$$\frac{\sqrt{2}}{5\sqrt{3}} = \frac{\sqrt{2}}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{5(3)} = \frac{\sqrt{6}}{15} \text{ or } \frac{1}{15}\sqrt{6}$$

Practice problems. Rationalize the denominator in each of the following:

1. $\frac{6}{\sqrt{2}}$
2. $\frac{\sqrt{5}}{\sqrt{3}}$
3. $\frac{2}{\sqrt{6}}$
4. $\frac{6}{\sqrt{y}}$

Answers:

1. $3\sqrt{2}$
2. $\frac{\sqrt{15}}{3}$
3. $\frac{\sqrt{6}}{3}$
4. $\frac{6\sqrt{y}}{y}$

EVALUATING RADICALS

Any radical expression has a decimal equivalent which may be exact if the radicand is a rational number. If the radicand is not rational, the root may be expressed as a decimal approximation, but it can never be exact. A procedure similar to long division may be used for calculating square root and cube root, and higher roots may be calculated by means of methods based on logarithms and higher mathematics. Tables of powers and roots have been calculated for use in those scientific fields in which it is frequently necessary to work with roots.

SQUARE ROOT PROCESS

The arithmetic process for calculation of square root is outlined in the following paragraphs:

1. Begin at the decimal point and mark the number off into groups of two digits each, moving both to the right and to the left from the decimal point. This may leave an odd digit at the right-hand or left-hand end of the number, or both. For example, suppose that the number whose square root we seek is 9025. The number marked off as specified would be as follows:

$$\sqrt{90'25.}$$

2. Find the greatest number whose square is contained in the left-hand group (90). This number is 9, since the square of 9 is 81. Write 9 above the first group. Square this number (9), place its square below the left-hand group, and subtract, as follows:

$$\begin{array}{l} \phantom{\sqrt{}}9 \\ \sqrt{90'25.} \\ \phantom{\sqrt{}}\underline{81} \\ \phantom{\sqrt{}}\ \ 9\ 25 \end{array}$$

Bring down the next group (25) and place it beside the 9, as shown. This is the new dividend (925).

3. Multiply the first digit in the root (9) by 20, obtaining 180 as a trial divisor. This trial divisor is contained in the new dividend (925) five times; thus the second digit of the root appears to be 5. However, this number must be added to the trial divisor to obtain a "true divisor." If the true divisor is then too large to use with the second quotient digit, this digit must be reduced by 1. The procedure for step 3 is illustrated as follows:

$$\begin{array}{rl} & \phantom{\sqrt{}}9\quad 5. \\ & \sqrt{90'25.} \\ & \phantom{\sqrt{}}\underline{81} \\ \cancel{180} & \phantom{\sqrt{}}\ \ 9\ 25 \\ 185 & \phantom{\sqrt{}}\ \ \underline{9\ 25} \\ & \phantom{\sqrt{}}\ \ 0\ 00 \end{array}$$

The number 180, resulting from the multiplication of 9 by 20, is written as a trial divisor beside the new dividend (925), as shown. The quotient digit (5) is then recorded and the trial divisor is adjusted, becoming 185. The trial quotient (180) is crossed out.

4. The true divisor (185) is multiplied by the second digit (5) and the product is placed below the new dividend (925). This step is shown in the illustration for step 3. When the product in step 4 is subtracted from the new dividend, the difference is 0; thus, in this example, the root is exact.

5. In some problems, the difference is not 0 after all of the digits of the original number have been used to form new dividends. Such problems may be carried further by adding 0's on the right-hand end of the original number, just as in normal long division. However, in the square root process the 0's must be added and used in groups of 2.

Practice problems. Find the square root of each of the following numbers:

1. 9.61 2. 123.21 3. 0.0025

Answers:

1. 3.1 2. 11.1 3. 0.05

TABLES OF ROOTS

The decimal values of square roots and cube roots of numbers with as many as 3 or 4 digits can be found from tables. The table in appendix I of this course gives the square roots and cube roots of numbers from 1 to 100. Most of the values given in such tables are approximate numbers which have been rounded off.

For example, the fourth column in appendix I shows that $\sqrt{72} = 8.4853$, to 4 decimal places. By shifting the decimal point we can obtain other square roots. A shift of two places in the decimal point in the radicand corresponds to a shift of one place in the same direction in the square root.

The following examples show the effect, as reflected in the square root, of shifting the location of the decimal point in the number whose square root we seek:

$$\sqrt{72} = 8.4853$$
$$\sqrt{0.72} = 0.84853$$
$$\sqrt{0.0072} = 0.084853$$
$$\sqrt{7,200} = 84.853$$

Cube Root

The fifth column in appendix I shows that the cube root of 72 is 4.1602. By shifting the decimal point we immediately have the cube roots of certain other numbers involving the same digits. A shift of three places in the decimal point in the radicand corresponds to a shift of one place in the same direction in the cube root.

Compare the following examples:

$$\sqrt[3]{72} = 4.1602$$
$$\sqrt[3]{0.072} = 0.41602$$
$$\sqrt[3]{72,000} = 41.602$$

Many irrational numbers in their simplified forms involve $\sqrt{2}$ and $\sqrt{3}$. Since these radicals occur often, it is convenient to remember their decimal equivalents as follows:

$$\sqrt{2} = 1.4142 \text{ and } \sqrt{3} = 1.7321$$

Thus any irrational numbers that do not contain any radicals other than $\sqrt{2}$ or $\sqrt{3}$ can be converted to decimal forms quickly without referring to tables.

For example consider

$$\sqrt{72} = 6\sqrt{2} = 6(1.4142) = 8.485$$
$$\sqrt{27} = 3\sqrt{3} = 3(1.7321) = 5.196$$

Keep in mind that the decimal equivalents of $\sqrt{2}$ and $\sqrt{3}$ as used in the foregoing examples are not exact numbers and the results obtained with them are approximate in the fourth decimal place.